A COMPLETE AND UNAMBIGUOUS SOLUTION TO THE SUPER-TSD MULTIPORT-CALIBRATION PROBLEM

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Abstract

A rigorous and complete mathematical analysis of the projective matrix transformation has led to a self-consistent and unambiguous solution to the Super-TSD multiport-calibration problem. Contrary to the conclusions of earlier analyses, the new procedure requires, in general, more than three n-port calibration-standards and determines the chain-scattering matrix (T-matrix) of the 2n-port Super-TSD virtual error-network up to an arbitrary scalar factor.

A generalization of the well known TSD Network-Analyzer calibration method was introduced in 1977[1,2] as the Super-TSD method. In contrast to the earlier TSD method, limited to two-port/zero-leakage situations, Super-TSD uses a single 2n-port virtual error-network as a comprehensive model for all types of calibration errors of a hypothetical multiport Network Analyzer.

The ability of the Super-TSD matrix-algorithm to remove overwhelming leakage errors was dramatically demonstrated in 1978[3,4] by computer simulation. Subsequently[5], a practical configuration was described for a multiport Network Analyzer (Figure 1), rigorously consistent with the earlier introduced Super-TSD error-model (Figure 2).

Indeed, in the appendix of reference[5], the relation between the uncalibrated and the calibrated multiport scattering matrices S_M and S_X was proved to be, for this configuration, expressed by the projective matrix transformation:

$$S_M = \left(T_1 \cdot S_X + T_2\right) \left(T_3 \cdot S_X + T_4\right)^{-1} \qquad (1)$$

where the T_i's (i = 1,...,4) are the n x n blocks of the 2n x 2n complex T-matrix of the Super-TSD 2n-port virtual error-network (Figure 2).

As a result of continued computer simulation work[4] and of early application attempts[6,7], it became increasingly clear that the originally proposed use of only three n-port calibration standards S_{Si} (i = 1, 2, 3), generating the corresponding uncalibrated readings S_{Mi}, was inadequate to sufficiently characterize the 2n-port Super-TSD error-network.

It was concluded that none of the four T-matrix blocks T_i (i = 1, ...,4) could be considered completely arbitrary and used as the basis for computing the remaining three blocks.

A recently developed, rigorous and complete mathematical analysis of the fundamental projective matrix-transformation (1) has shown that any error-network T-matrix, consistent with only three n-port standards S_{Si} (physically implemented as different combinations of "Throughs," "Shorts," and "Delays") and their corresponding uncalibrated readings S_{Mi} (i = 1, 2, 3), must have the structure:

$$T = \left|\begin{array}{c|c} T_1 & T_2 \\ \hline T_3 & T_4 \end{array}\right| = \left|\begin{array}{c|c} M_1 & M_2 \\ \hline M_3 & M_4 \end{array}\right| \cdot \left|\begin{array}{c|c} F & 0 \\ \hline 0 & F \end{array}\right| \cdot \left|\begin{array}{c|c} S_1 & S_2 \\ \hline S_3 & S_4 \end{array}\right| \qquad (2)$$

where the M_j and S_j are determined by the measurements, and the n x n arbitrary complex matrix-block F is impossible to determine on the basis of the given S_{Si}, S_{Mi} information only.

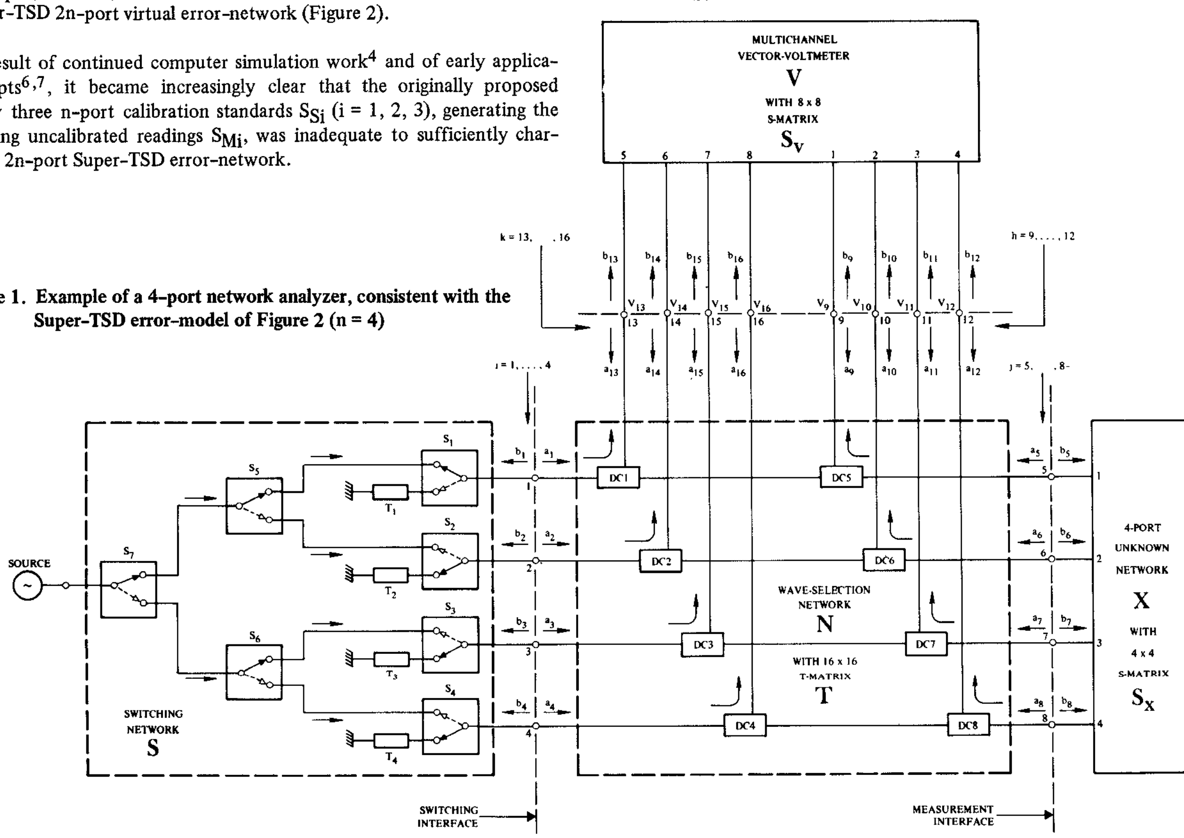


Figure 1. Example of a 4-port network analyzer, consistent with the Super-TSD error-model of Figure 2 (n = 4)

Specifically, the n x n matrices S_j, in the matrix-structure (2), are the four matrix-parameters of the cross-ratio-like projective matrix-transformation:

$$Y = \left(S_1 \cdot X + S_2\right)\left(S_3 \cdot X + S_4\right)^{-1} =$$
$$= \left(S_{S1} - S_{S2}\right)\left(S_{S2} - S_{S3}\right)^{-1}\left(S_{S3} - X\right)\left(X - S_{S1}\right)^{-1} \quad (3)$$

that maps the three calibration standards $X = S_{Si}$ to the three "mathematical" standards $\infty \cdot I$, I, $0 \cdot I$ (the infinite, identity and zero matrices).

Similarly, the n x n matrices M_j are the four matrix-parameters of the projective matrix-transformation:

$$W = \left(M_1 \cdot Z + M_2\right)\left(M_3 \cdot Z + M_4\right)^{-1} \quad (4)$$

that maps the mathematical standards $\infty \cdot I$, I and $0 \cdot I$ to the uncalibrated readings S_{Mi} (i = 1, 2, 3). This transformation is the inverse of the cross-ratio-like transform:

$$Z = \left(S_{M1} - S_{M2}\right)\left(S_{M2} - S_{M3}\right)^{-1}\left(S_{M3} - W\right)\left(W - S_{M1}\right)^{-1} \quad (5)$$

The reason for the arbitrariness of the matrix-block F is that any scalar matrix Y_S, generated by the transform (3), maps into itself through the reduced projective transformation:

$$Y_S = F \cdot Y_S \cdot F^{-1} \quad (6)$$

More than three calibration standards are thus required to determine the matrix-block F and this computation only leads to a determination of F up to an arbitrary scalar factor. This is, however, consistent with the invariance of the transformation (1) with respect to scalar scaling of the T-matrix of the 2n-port error-network.

The determination of the matrix-block F may be performed by first computing the matrices Y_j that are the images of the additional standards S_{Sj} (j = 4,5...), generated by (3) and the matrices Z_j that are the images of the additional uncalibrated readings S_{Mj}, generated by (5):

$$Y_j = \left(S_{S1} - S_{S2}\right)\left(S_{S2} - S_{S3}\right)^{-1}\left(S_{S3} - S_{Sj}\right)\left(S_{Sj} - S_{S1}\right)^{-1} \quad (7)$$

$$Z_j = \left(S_{M1} - S_{M2}\right)\left(S_{M2} - S_{M3}\right)^{-1}\left(S_{M3} - S_{Mj}\right)\left(S_{Mj} - S_{M1}\right)^{-1} \quad (8)$$

and observing then that:

$$Z_j = F \cdot Y_j \cdot F^{-1} \quad (9)$$

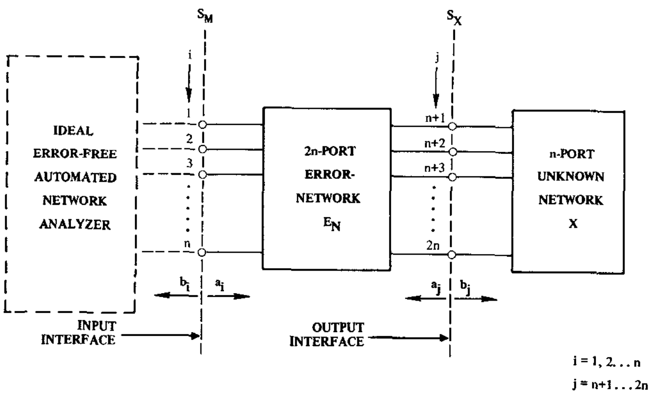
or:

$$Z_j \cdot F - F \cdot Y_j = 0 \quad (9')$$

This is a well known type of matrix-equation[8]. A condition for the existence of a nonzero solution is that Z_j and Y_j have at least one common eigenvalue. If the matrix F must be invertible, then all the eigenvalues of Z_j must equal those of Y_j.

There are N linearly independent solutions for each of the additional equations type (9'), where N is a summation of all the degrees of the greatest common divisors of the elementary divisors of the matrices Z_j and Y_j. It has been proved that no more than two independent equations of the type (9') are necessary and at the same time sufficient to uniquely determine the matrix-block F, up to an arbitrary scalar factor.

This result implies that a minimum of five n-port calibration standards are required to fully characterize the Super-TSD 2n-port error-network, in the general case. In addition, five standards actually determine the Super-TSD 4-port error network in the n = 2 case.



$$\begin{bmatrix} b_i \\ a_i \end{bmatrix} = \begin{bmatrix} T_1 & T_2 \\ T_3 & T_4 \end{bmatrix} \cdot \begin{bmatrix} a_j \\ b_j \end{bmatrix} \qquad \begin{bmatrix} a_{n+1} \\ \vdots \\ a_{2n} \end{bmatrix} = S_X \cdot \begin{bmatrix} b_{n+1} \\ \vdots \\ b_{2n} \end{bmatrix} \qquad \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} = S_M \cdot \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}$$

$$S_M = (T_1 \cdot S_X + T_2)(T_3 \cdot S_X + T_4)^{-1}$$

Figure 2. The generalized Super-TSD-error-model is a virtual 2n-port error-network E_N. This network is assumed to be always connected between the unknown n-port network X and an ideal error-free n-port automated network analyzer. The n x n matrices $T_1 \ldots T_4$ are the quadrants of the 2n x 2n T-matrix T of the error-network E_N.

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